

# Parameters Affecting Period of Vibration of Symmetrical Reinforced Concrete Buildings in Seismic Analysis

PRAKASH SANGAMNERKAR<sup>1</sup>,

*Research Scholar, Maulana Azad National Institute of Technology, Bhopal (M.P.), India,  
nerkar17@rediffmail.com*

Dr. S. K. DUBEY<sup>2</sup>

*Professor, Maulana Azad National Institute of Technology, Bhopal (M.P.), India,  
dubeysk2000@yahoo.com*

## Abstract

Indian seismic codes provide empirical expressions to evaluate fundamental period of vibration of reinforced concrete (RC) buildings to derive design base shear. Empirical equation prescribed in the seismic design code is a function of height of the building alone and do not consider the effects of other structural parameters of the building. Fundamental period predicted by these expressions are widely used in the practice. It is therefore very important to use realistic values of time period in seismic design of the structures. This paper deals with the characterization of parameters which affects fundamental period of vibration of symmetrical RC buildings and derive improved period equation which incorporates the parameters like number of bays in either direction, base width / plan area of the building and stiffness of the structure, in addition to the height of the structure. Period equations were developed by performing nonlinear regression analysis on the results obtained by dynamic analysis of 180 various structural configurations.

**Key Words:** Fundamental Period, Nonlinear regression analysis, Number of Bays, Dynamic Analysis, Stiffness of the structure.

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<sup>1</sup> Research Scholar Deptt. of Civil Engineering, Maulana Azad National Institute of Technology, Bhopal (M.P.) 462051, 09406912035, e-mail: [nerkar17@rediffmail.com](mailto:nerkar17@rediffmail.com).

<sup>2</sup> Professor, Deptt. Of Civil Engineering, Maulana Azad National Institute of Technology, Bhopal (M.P.) 462051, 09424455828, e-mail: [dubeysk2000@yahoo.com](mailto:dubeysk2000@yahoo.com)

## 1 Introduction

The fundamental period of vibration depends upon the distribution of mass and stiffness along the height of the building and estimation of seismic base shear requires the fundamental natural period of vibration  $T$  of the building. Seismic induced dynamic forces and its directions vary substantially with time cause considerable inertia forces on the buildings. However for the building configuration adopted and the construction material chosen, it is not always possible to exactly determine from theoretical considerations, that is, through detailed dynamic analysis. Hence, empirical formulae obtained through experimentally observed behavior of buildings are utilized [1-3]. The stiffness contribution of many non structural elements, such as in-fill masonry panels [4, 5] also considered to derive period formula in different countries. For this reason, the empirical expression for  $T$  may be specific to each country. The approximate fundamental natural period of vibration ( $T_a$ ) in second of a moment resisting frames building without brick infill panels may be estimated by empirical expression given in Indian seismic code IS1893 (Part-1)-2002 [6]

$$T_a = 0.075h^{0.75} \quad \text{for R.C. frame building} \quad (1.1)$$

$$= 0.085h^{0.75} \quad \text{for steel frame building} \quad (1.2)$$

$$T_a = 0.09 h / \sqrt{d} \quad \text{for all other buildings.} \quad (1.3)$$

Behavior of any structure under dynamic forces depends upon the dynamic characteristics of structures, which are controlled by both their mass and stiffness properties; in addition, the performance of structures also depends on the number of bays in either direction along with the plan area of the building. The fundamental period of buildings can be determined by an exact eigenvalue analysis, or by a rational method called Rayleigh's method, with the use of computer solutions. The fundamental period determined by these methods is usually longer than the period obtained by code equations.

The fundamental period can be evaluated using simplified expressions 1.1 to 1.3 found in codes, which are based on earthquake recordings in existing buildings, laboratory tests, numerical or analytical computations. These technical codes provide expressions which depend on basic parameters such as building height or number of stories. Building periods predicted by these expressions are widely used in practice although it has been pointed out by Amanat and

Hoque [7] and Verderame, Iervolino and Manfredi [8] that there is scope for further improvement in these equations since the height alone is inadequate to explain period variability.

The aim of this study is to characterize parameters which affects fundamental period of vibration of symmetrical RC buildings and derive improved period equation which incorporates the parameters like number of bays in direction, base width / plan area of the building and stiffness of the structure, in addition to the height of the structure.

## 2 Literature study

Since the predicted fundamental period is used to obtain the expected seismic load affecting the structure, a precise estimation of it is important for the safety of the applied procedure in the design steps and consequently in the future performance of the structure after it is constructed. The fundamental period of vibration required for the simplified design of RC structures has been calculated for many years using a simplified formula relating the period to the height of the building.

Gerardo M. [8] pointed out that height alone seems inadequate to explain period variability and the results of this study suggest that. Also a global parameter (e.g., plan area) should be added in simplified relationships for rapid period evaluation. Therefore, an expression which includes also the plan area is considered in the following equations.

$$T = \alpha H^{\beta} S^{\gamma} \quad (2.1)$$

where S is the product of the two principal plan dimensions of the building Lx and Ly.

M. Hadzima et. al [9] seven different equations proposed in their study in order to determine more accurate expressions for the elastic period they considered seven basic expressions which, in addition to the number of floors, take into consideration each of the following:

- The number of bays parallel to the considered direction;
- The ratio between the number of bays in the longitudinal and transversal directions;
- The product between the number of bays in the longitudinal and transversal directions.

Following are the expressions proposed to evaluate period of vibration:

$$T = C_1 N^{C_2} \quad (2.2)$$

$$T = C_1 N^{C_2} B^{C_3} \quad (2.3)$$

$$T = C_1 N^{C_2} + C_3 B^{C_4} \quad (2.4)$$

$$T = C_1 N^{C_2} \left( \frac{B_x}{B_y} \right)^{KC_3} \quad (2.5)$$

$$T = C_1 N^{C_2} (B_x \cdot B_y)^{C_3} \quad (2.6)$$

$$T = C_1 N^{C_2} + C_3 (B_x \cdot B_y)^{C_4} \quad (2.7)$$

$$T = C_1 N^{C_2} + C_3 \left( \frac{B_x}{B_y} \right)^{C_4} \quad (2.8)$$

Kwon and Kim [10] present an extensive review of the evolution of the code equations from the 1970s to 2010. In addition, the authors conducted a quantitative comparison of measured fundamental period and estimated fundamental periods calculated from the code equations. It was found that when looking at buildings between 6 and 8 stories, the difference between the code approximation and the measured period is relatively large. This leads the authors to conclude that further refinement to these equations is needed depending on building heights without actually suggesting any improvement.

Further in the literature survey it is observed that many authors [11-16] have proposed fundamental natural period of vibration in the form of:

$$T = C_1 \cdot H^{C_2} \quad (2.9)$$

Similarly some of the authors [17- 21] proposed expression of the form:

$$T = C_1 \cdot H \quad (2.10)$$

Where value of C1 and C2 derived by regression analysis using different mathematical model by corresponding authors based on their experimental results, actual recordings on instrumented buildings, or using computer simulation.

### 3. Parameters for Analysis

Various RC frame buildings were analyzed using computerized solution with the following assumption mentioned in Table 3.1 to Table 3.3 and general arrangement of beams and columns are depicted in Figure 3.1.

Table 3.1 Building configuration

Type of structure	Multistory rigid jointed plane frames
No of storey	GF to G+11, G+24.
Floor height	3.6m
No of Grids	2x6, 3x6, 4x6, 5x6, 3X3, 4X4, 6x6, 8x8.

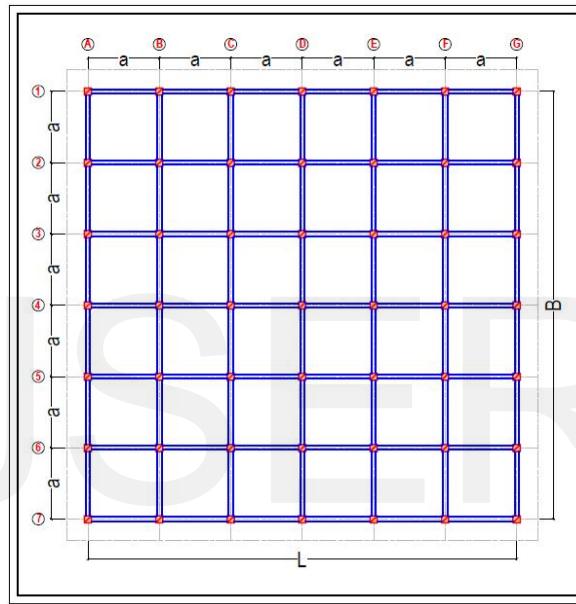


Figure 3. 1 Typical Plan with 6x6 grid

Table 3.2 Materials

Material used	Concrete M-25 and Reinforcement Fe-415.
Type of soil	Type -II, Medium soil as per IS-1893
$E_c$	$5000\sqrt{f_{ck}} \text{ N/ mm}^2$
$F_{cr}$	$0.7\sqrt{f_c} \text{ k N/ mm}^2$

Table 3.3 Structural details

Size of columns	1.0x1.0m, 0.75x0.75m, 0.6x0.6m, 0.5x0.5m., 0.4x0.4m, 0.3x0.3m.
Thickness of slab	150mm
Walls- (a) External	200 mm
(b) Internal	100 mm
Imposed load [21]	4.00kN/ m <sup>2</sup>
Floor finish	1.00kN/ m <sup>2</sup>
Water proofing	2.500kN/ m <sup>2</sup>
Specific wt. of RCC	25.00 kN/ m <sup>3</sup>

As mentioned in Tables 3.1-3.3 and Figure 3.1, total 180 values of the time period were obtained by performing dynamic analysis on various building / structural configurations, earthquake parameters for the buildings were considered as per the provisions made in the Indian seismic code for earthquake zone III, computer software STAAD [23] is used to analyze the building models. To get the exact value of structural response of earthquake excitation, the response contribution of all natural modes of vibration have been included; generally first few modes are usually sufficient to arrive accurate results. In the present investigation, 20 modes of vibration are considered to get 90 % mass participation.

#### 4. PROPOSED EQUATIONS

Table 4.1 shows the value of fundamental period of vibration for 60 various building configurations having grid length of 4m, 6m & 8m. For each grid length various values for fundamental period are tabulated for column sizes 1000x1000mm to 500x500mm.

Table No. 4.1 Fundamental Period of Vibration (sec) for Grid length 4 m, 6m, and 8m Grid Length

Column Size (mm)												
	1000x1000			750x750			600x600			500x500		
	Grid Length (m)			Grid Length (m)			Grid Length (m)			Grid Length (m)		
	4.00	6.00	8.00	4.00	6.00	8.00	4.00	6.00	8.00	4.00	6.00	8.00
2X6	4.46	5.92	7.90	4.78	6.23	8.30	5.33	6.75	8.85	6.04	7.48	9.63
3x6	4.09	5.71	7.76	4.31	5.96	8.12	4.75	6.42	8.64	5.38	7.11	9.41
4x6	3.91	5.60	7.69	4.07	5.82	8.03	4.45	6.26	8.54	5.04	6.93	9.31
5x6	3.80	5.54	7.65	3.93	5.75	7.98	4.28	6.16	8.48	4.84	6.83	9.25
6x6	3.73	5.50	7.62	3.84	5.70	7.95	4.18	6.11	8.45	4.72	6.77	9.22

Table 4.1 shows the value of fundamental period of vibration for 60 various building configurations having grid length of 4m, 6m & 8m. For each grid length various values for fundamental period are tabulated for column sizes 1000x1000mm to 500x500mm. Table 4.1 shows that for 2x6 grid values of period of vibration comes out to be 4.46 sec, whereas 3.73 seconds for 6x6 grid size building, in the same table period of vibration observed to be 4.46 sec for the column size 1000x1000mm and 6.04 sec for column size 500x500mm. Similar results are also tabulated in succeeding columns of table for column sizes 750x750mm, 600x600mm and 500x500mm. Values so obtained are compared with square shaped building i.e. 6x6 grid, which is considered as a base case for comparing the results, it can be observed that the rectangular shaped buildings observed to be having greater period of vibration as compared to square shaped buildings and period of vibration of rectangular building having 19.39 % higher as compared to square shaped building, for the same building as in Table 4.1. Period of vibration observed to be increasing as the column sizes are reducing. Hence by reducing the column sizes, period of vibration is increased.

On the basis of analysis performed on 60 square/rectangular shaped buildings, it is observed that, evaluation of period of vibration of buildings, depends upon the base dimensions, number of grids and stiffness of the structure, and the contribution of the same should also be incorporated in the formula prescribed to evaluate the fundamental time period of the building in seismic analysis.

To develop period formula including parameters like number of bays, bay width, plan area of the building, stiffness of the structure another building configurations with above variables including height of the building as one of the parameter, analyzed and obtained total 120 results, which are tabulated in the following tables i.e. Tables 4.2 - 4.5

Table No 4.2 Values of Time Period in Seconds 6 Bays

S. No.	Number of storey	Height (m)	Column Size (mm)		
			500X500	400X400	300X300
1	G+11	43.2	2.50019	3.09978	4.62162
2	G+10	39.6	2.28882	2.83916	4.23605
3	G+9	36	2.07895	2.58042	3.8529
4	G+8	32.4	1.8705	2.32352	3.47204
5	G+7	28.8	1.66343	2.06836	3.09341
6	G+6	25.2	1.45764	1.81489	2.71693
7	G+5	21.6	1.2531	1.56303	2.34255
8	G+4	18	1.07182	1.34166	2.01599
9	G+3	14.4	0.84763	1.064	1.60005
10	G+2	10.8	0.64689	0.81694	1.23216
11	G+1	7.2	0.4926	0.62921	0.95607
12	GF	3.6	0.25502	0.33188	0.5097

Table No 4.3 Values of Time Period in Seconds 8 Bays

S. No.	Number of storey	Height (m)	Column Size (mm)		
			500X500	400X400	300X300
1	G+11	43.2	2.03604	2.58332	3.94172
2	G+10	39.6	1.76833	2.24024	3.41531
3	G+9	36	1.60478	2.03438	3.10448
4	G+8	32.4	1.44259	1.83022	2.79569
5	G+7	28.8	1.28171	1.62767	2.48893
6	G+6	25.2	1.2206	1.42669	2.18412
7	G+5	21.6	1.01446	1.29384	1.98575
8	G+4	18	0.80624	1.02915	1.58018
9	G+3	14.4	0.65001	0.83253	1.28102
10	G+2	10.8	0.495	0.63739	0.98385
11	G+1	7.2	0.34178	0.44414	0.68924
12	GF	3.6	0.19346	0.25549	0.40057



Table No. 4.4 Fundamental Period of Vibration ( sec ) for Grid length 7 m

Base Width 42.0x42.0m					
Number of Storeys	Height (m)	Column Size (mm)			
		1000x1000	750x750	600x600	500x500
G+20	77.6	5.41473	5.65868	6.04156	6.64024
G+16	63.2	7.3172	4.55074	4.87107	5.35712
G+12	48.8	3.23033	3.45621	3.71917	4.0977
G+8	34.4	2.15944	2.37393	2.58371	2.85953
G+4	20	1.12596	1.30494	1.46103	1.63868
GF	5.6	0.21027	0.29016	0.35746	0.4255

Table No. 4.5 Fundamental Period of Vibration ( sec ) for Grid length 8 m

Base Width 48.0x48.0m					
Number of Storeys	Height (m)	Column Size (mm)			
		1000x1000	750x750	600x600	500x500
G+20	77.6	6.32883	6.64155	7.06875	7.71718
G+16	63.2	5.04644	5.34748	5.70986	6.23949
G+12	48.8	3.77494	4.06541	4.36748	4.78322
G+8	34.4	2.52194	2.79432	3.0397	3.34619
G+4	20	1.61209	1.53439	1.72038	1.92234
GF	5.6	0.24025	0.3367	0.41701	0.49595

Since objective of this paper is to characterize parameters which affects fundamental period of vibration of symmetrical RC buildings and develop a simplified equation to allow design engineers to quickly and accurately estimate the fundamental period of moment resisting framed structures by taking into account not only the height (H), but also the base dimension /plan area of the building, stiffness of the structure and number of bays in longitudinal and transvers direction.

Proposed equations will give more accurate period values than the current code equations. The proposed equation is developed by multiple nonlinear regression analysis on the results of dynamic analysis tabulated in Tables 4.2-4.5. The program SPSS v16 is used to perform the regression analysis. Several equation forms (Equations 4.1 - 4.4) were investigated, including power models of varying form, quadratic models, polynomial models, and linear models. The equations were modified including structural parameters in varying ways, such as height, ratio of

total cross sectional area of columns to total plan area of the building, number of bays in longitudinal direction and number of bays in transvers direction.

$$T = C_1 \cdot (N_b \cdot N_l)^{C_2} \cdot H^{C_3} \tag{4.1}$$

$$T = C_1 \cdot \left(\frac{N_b}{N_l}\right)^{C_2} \cdot H^{C_3} \tag{4.2}$$

$$T = C_1 \cdot (A)^{C_2} \cdot H^{C_3} \tag{4.3}$$

$$T = C_1 \cdot (N_b \cdot N_l)^{C_2} \cdot \left(\frac{A_c}{A_b}\right)^{C_3} \cdot H^{C_4} \tag{4.4}$$

This equation form will be referred to as a 3-variable power model. The four regression parameters are obtained by minimizing the sum of the squares of the distances between the actual data points and the regression curve. For each regression, the standard deviation of the residuals ( $\sigma$ ) and r-squared ( $R^2$ ) values are found. These values indicate how well the regression equation fits the sample Rayleigh data.

Consequently, equation 4.4 with parameters  $C_1=0.005$ ,  $C_2=-0.061$ ,  $C_3=-0.55$  and  $C_4=1.049$  is proposed as the best-fit equation for determining the approximate fundamental period of moment resisting frames. In this study, a value of  $R^2=0.945$  indicates that 94.5% of the variation in the dependent variable is explained by the regression model.

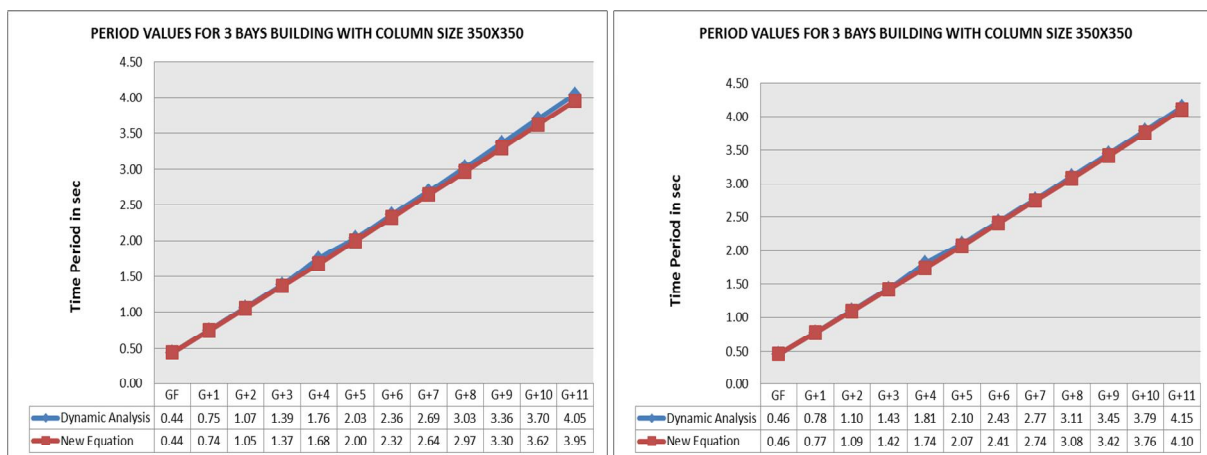


Figure 4. 1 Comparison of period values (3 bays)

Figure 4. 2 Comparison of period values (4 bays)

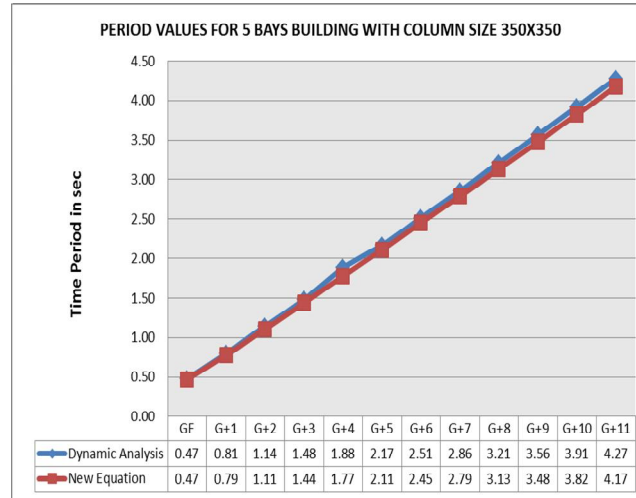


Figure 4. 3 Comparison of period values (5 bays)

## 5. CONCLUSION

Regression analysis was performed on total 180 results obtained from the dynamic analysis and Equations 4.1–4.4 were derived. For validation of the new equations, another 36 building configurations were considered and calculated the value of time period using eq 4.4, values so derived were compared by performing dynamic analysis. It is observed from the charts (Figures 4.1-4.3) that the values of time periods from new equation are almost matching with the values derived in the dynamic analysis. and the variation observed to be in between -3% to 10%. Therefore, height alone seems inadequate to evaluate period of vibration and the results of this study suggest that plan area of the building, number of bays in either direction and stiffness of the structure should also be added in simplified relationships for evaluation of time period of vibration in seismic analysis.

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